King's College London

University Of London

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

ATTACH this paper to your script **USING THE STRING PROVIDED**

Candidate No: ..... **Desk No:** .....

MSc Examination

7CCMFM06 Numerical and Computational Methods in Finance

Summer 2016 Resit Examination

Time Allowed: Two Hours

All questions carry equal marks. Full marks will be awarded for complete answers to FOUR questions. Only the best FOUR questions will count towards grades A or B, but credit will be given for all work done for lower grades.

NO calculators are permitted.

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

TURN OVER WHEN INSTRUCTED

1. A trader has P units of cash and wishes to invest in a stock and a risk free bond to maximize their expected utility at time T. The trader's utility function is

$$u(x) = \begin{cases} \ln(x) & \text{if } x > 0, \\ -\infty & \text{otherwise.} \end{cases}$$

The trader believes the stock follows geometric Brownian motion

$$\mathrm{d}S_t = S_t(\mu\,\mathrm{d}t + \sigma\,\mathrm{d}W_t).$$

The bond has interest rate r. At time 0 the trader invests an amount Q of their wealth in stock and the rest in bonds.

(i) Write the expected utility as an integral. [**40%**]

(ii) Write the MATLAB code to compute this integral by a Monte Carlo method. [**30%**]

(iii) State a variance reduction technique you could use to improve the rate of convergence of the Monte Carlo method. [**10%**]

(iv) $u(x)$ takes the value $-\infty$ when x is negative. What trading constraint does this imply? [**10%**]

(v) How could you use MATLAB to find the optimal value of Q? [**10%**]

2. (i) What is meant by VaR and CVaR? [**20%**]

(ii) What is the sub-additivity property of a coherent risk measure? Show that VaR is not sub-additive. [**20%**]

(iii) Write a difference equation you could use to simulate a stock price that follows the Black–Scholes model. [**20%**]

(iv) Describe how you could use the results of such a simulation to estimate the VaR of a call option on a stock. [**20%**]

(v) Explain briefly how you could test the results of this calculation. [**20%**]

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3. (i) Write down the difference equations for the implicit finite difference scheme for the Black–Scholes PDE

$$V_t + \frac{1}{2}\sigma^2 S^2 V_{SS} + rSV_S - rV = 0.$$

[**30%**]

(ii) Explain how these difference equations can be used in practice to find a numerical solution if appropriate boundary conditions are given. [**20%**]

(iii) Explain why the implicit scheme may be preferable to the explicit scheme for practical computations. [**20%**]

(iv) A trader has created a new financial derivative. It is a derivative on a stock price S_t. The derivative pays out the amount S_t^2 at the first moment that the stock price exceeds $C(1-t)$ where t is the time since the contract was sold. C is some constant.

You wish to price this derivative in the Black–Scholes model. What boundary conditions would you use to price this derivative using the implicit scheme above? [**30%**]

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4. Let A be a real symmetric, positive definite matrix. Let L denote the Cholesky decomposition of A. It can be computed using the formulae

$$L_{j,j} = \sqrt{A_{j,j} - \sum_{k=1}^{j-1} L_{j,k}^2},$$
$$L_{i,j} = \frac{1}{L_{j,j}} \left(A_{i,j} - \sum_{k=1}^{j-1} L_{i,k} L_{j,k} \right) \qquad \text{when } i > j.$$

This means we can compute the entry $L_{i,j}$ if we know all the entries to the left and above.

(i) What should be the value of $L_{i,j}$ when $i < j$? [**10%**]

(ii) Write a Matlab function to compute the Cholesky decomposition of a matrix A. Your function should proceed by computing all the values of $L_{i,j}$ for each row i before moving onto the next row. [**50%**]

(iii) Write an automated test for your function. You may assume you have already written a function `assertApproxEquals` as described in the lectures.

[**30%**]

(iv) What will happen if A is not positive definite? [**10%**]

5. (i) Working in the Black–Scholes model, explain how you could price an Asian call option with strike K and maturity T using the Monte Carlo method. Be sure to describe:

(a) the stock price process you will simulate and how it is simulated will simulate it;

[**20%**]

(b) the definition of an Asian option and how this effects your simulation;

[**20%**]

(c) how you will compute a price from your simulation; [**20%**]

(d) how you would test your answer is correct. [**20%**]

(ii) Briefly explain how would you compute the delta of this option. [**20%**]

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