Worksheet 4

There is a quiz to test basic understanding available on Keats.

1) [$\star$] What tests could you write for the `generateBSPaths` function? Write them and check that they pass. Make sure that the tests always pass whatever numbers are generated.

(Solution: see the file `testGenerateBSPaths.m` in `lecture4.zip`)

2) Use the `generateBSPaths` function to create a plot of the percentiles of the stock price over time as shown in the slides.

(Solution: see the file `plotBSPercentiles.m` in `lecture4.zip`)

3) [$\star$] Sketch a graph of the how the price of a knockout option varies as the barrier changes. What is the minimum value and when is it obtained? What is the maximum value and when is it obtained? Explain your answer. Check that your sketch matches the results of the Monte Carlo simulation.

(Solution: see the file `plotKnockoutAgainstBarrier.m` in `lecture4.zip`)

4) [$\star$] Price a knock in option and an Asian option. How can you use function passing to improve your code?

(Solution: see the file `testPriceDerivativeByMonteCarlo.m` in `lecture4.zip`)

5) For our Monte Carlo pricing algorithm we need to generate normally distributed random numbers. Suppose that we do this by generating a uniformly distributed random number and then applying N^{-1}. Show that the Monte Carlo pricing algorithm then becomes equivalent to one of the Monte Carlo integration techniques from the last lecture.

6) [$\star$] Prove the convergence rates for the forward, backward and central estimates for the derivative using Taylor's theorem assuming that f is continuously differentiable to all orders.

7) Refactor the code so that the random numbers used to compute the delta are not generated twice.

(Solution: see the file `testComputeDeltaByMonteCarlo2.m` in `lecture4.zip`)

8) We have stated an approximation formula for the second derivative in terms of the value of f at the points $\{x-h, x, x+h\}$. Assuming f is smooth, compute a bound on the error of this formula.

9) [$\star$] Use the approximation formula for the second derivative to write a function that computes the gamma of a European option using a Monte Carlo method. Check your answer.

(Solution: see the file `testComputeGammaByMonteCarlo.m` in `lecture4.zip`)

10) Numerically compute the derivative of $\sin(x)$ in two different ways and plot a log-log plot of the error in the estimates.

(Solution: see the file `plotDerivativeEstimates.m` in `lecture4.zip`)

11) [$\star$] Implement Monte Carlo pricing with antithetic sampling for a knockout option.

(Solution: see the file `testPriceKnockoutByMonteCarloAntithetic.m` in `lecture4.zip`)

12) [$\star\star$] May 2016, Q5