Worksheet 6

There is a quiz to test basic understanding available on Keats.

1) [$\star$] A stock follows the Black Scholes model with $S_0 = 1$, $\sigma = 0.2$ and $\mu = 0.08$. The risk free rate is 0.05. An investor has 1 dollar to invest for a time period of 1 year and wishes to optimize their expected utility. Their utility function is

$$u(x) = \begin{cases} \ln(x) & x > 0 \\ -\infty & x \leq 0 \end{cases}$$

Compute their expected utility:

(a) By the Monte Carlo method

(b) By the Monte Carlo method with antithetic sampling

(c) By the Monte Carlo method with a control variate of your choice

(d) Use the rectangle rule.

(e) Compare the errors of these approaches

2) [$\star$] You can compute the area of the unit circle using a Monte Carlo method. Simply generate uniformly distributed points in $[-1, 1] \times [1, 1]$ and count how many lie in the circle. Implement this in MATLAB.

Which is better using $2N$ uniformly distributed points in $[-1, 1] \times [-1, 1]$ or using $2N$ points generated using antithetic sampling? Explain your answer.

3) Use matlab's `chol` function to find the Cholseky decomposition of

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$$

(Solution: see the file `testRandnMultivariate.m` in `lecture6.zip`)

4) Use matlab to plot a scatter plot of 10000 points (X, Y) where X and Y are normally distributed with covariance matrix

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$$

(Solution: see the file `testRandnMultivariate.m` in `lecture6.zip`)

5) [$\star$] X is normally distributed with mean 5 and standard deviation 3 and Y is normally distributed with mean 7 and standard deviation 1 and if X and Y have correlation $\rho = 0.5$. Generate a sample of points (X, Y) matching these properties. How have you tested your answer?

(Solution: see the file `testQuestion3.m` in `lecture6.zip`)

6) What is the transformation matrix B that reverses the order of the coordinates x_1, x_2, x_3? What is BB^T? Use this to find a pseudo square root of the matrix:

$$\begin{pmatrix} 5 & 1 & 1 \\ 1 & 6 & 1 \\ 1 & 1 & 4 \end{pmatrix}$$

which is not upper triangular

(Solution: see the file `solutions.txt.m` in `lecture6.zip`)

7) [$\star$] Write a function `randnMultivariate(omega,n)` which generates `n` samples from a multivariate normal distribution with covariance matrix omega.

(Solution: see the file `testRandnMultivariate.m` in `lecture6.zip`)

8) Compute the Cholesky decomposition of

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$$

by hand.

9) [$\star$] Simulate the process

$$\mathrm{d}S_t = S_t(\mu \mathrm{d}t + \sigma \mathrm{d}W_t)$$

using the Euler scheme and find the exact solution too. Use this to generate a log-log plot of errors for the Euler scheme.

(Solution: see the file `plotStockPriceErrors.m` in `lecture6.zip`)

10) [$\star$] Simulate the Vasicek interest rate model

$$\mathrm{d}r_t = a(b - r_t)\mathrm{d}t + \sigma \mathrm{d}W_t$$

using the Euler scheme. Generate plots of interest rate paths with varying parameters so you get a feel for this kind of model. How could you simulate the Vasicek model without using the Euler scheme? (HINT: The increments of the Vasicek model over any time period are known to be normally distributed and there are formulae for their mean and variance)

(Solution: see the file `testSimulateVasicek.m` in `lecture6.zip`)

11) Modify the delta hedging code from last week so that one still delta hedges as though one believed the Black–Scholes model was true, but in fact the interest rates are stochastic and follow the Vasicek model. How does the delta hedging strategy perform?

12) [$\star\star$] Mock exam Q4

13) [$\star\star$] Bonus questions Q1

14) May 2015 Q3

15) May 2016 Q3

16) May 2016 Q4

17) Bonus questions Q4