

# Pension indexation and Intergenerational risk sharing in pay-as-you-go pension schemes

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# Outline

- 1. Motivation
- 2. Optimization model
- 3. Numerical illustration
- 4. Conclusions

## Reference :

Morsomme H., Alonso-Garcia J. , Devolder P. (2024):  
“Intergenerational risk sharing in pay-as-you-go pension schemes” , *Scandinavian Actuarial Journal* 2025(4), 404–432

# 1. Motivation

*A complicated combination for social security pension schemes in many countries :*

**DB + PAYG + Aging**

*3 main challenges to address simultaneously :*

- financial sustainability*
- social adequacy*
- fairness / equity*
  - inter generation*
  - intra generation*

# Dependency ratio

|           | 2022        | 2070        |
|-----------|-------------|-------------|
| Be        | <b>33,7</b> | <b>53,0</b> |
| NL        | 34,3        | 56,3        |
| D         | 37,4        | 55,0        |
| FR        | 38,2        | 57,8        |
| SE        | 36,0        | 50,4        |
| IT        | 40,8        | 65,5        |
| <b>EU</b> | <b>36,1</b> | <b>59,1</b> |



Demographic Dependency ratio = population 65+ / population 20-64

# PAYG Equilibrium Equation

## Incomes :

$A(t)$  = number of contributors at time  $t$

$S(t)$  = mean salary

$\pi(t)$  = (implicit) contribution rate



$$IN(t) = A(t) \cdot \pi(t) \cdot S(t)$$

# PAYG Equilibrium Equation

## Outcomes :

$B(t)$  = number of retirees at time  $t$

$P(t)$  = mean pension

$\delta(t)$  = benefit ratio =  $\frac{P(t)}{S(t)}$



$$\text{OUT}(t) = B(t).P(t) = B(t).\delta(t).S(t)$$

# PAYG Equilibrium Equation

Actuarial equilibrium :

$$\text{IN}(t) = \text{OUT}(t)$$



$$A(t) \cdot \pi(t) \cdot S(t) = B(t) \cdot P(t)$$

$$\pi(t) = \frac{B(t)}{A(t)} \cdot \frac{P(t)}{S(t)}$$

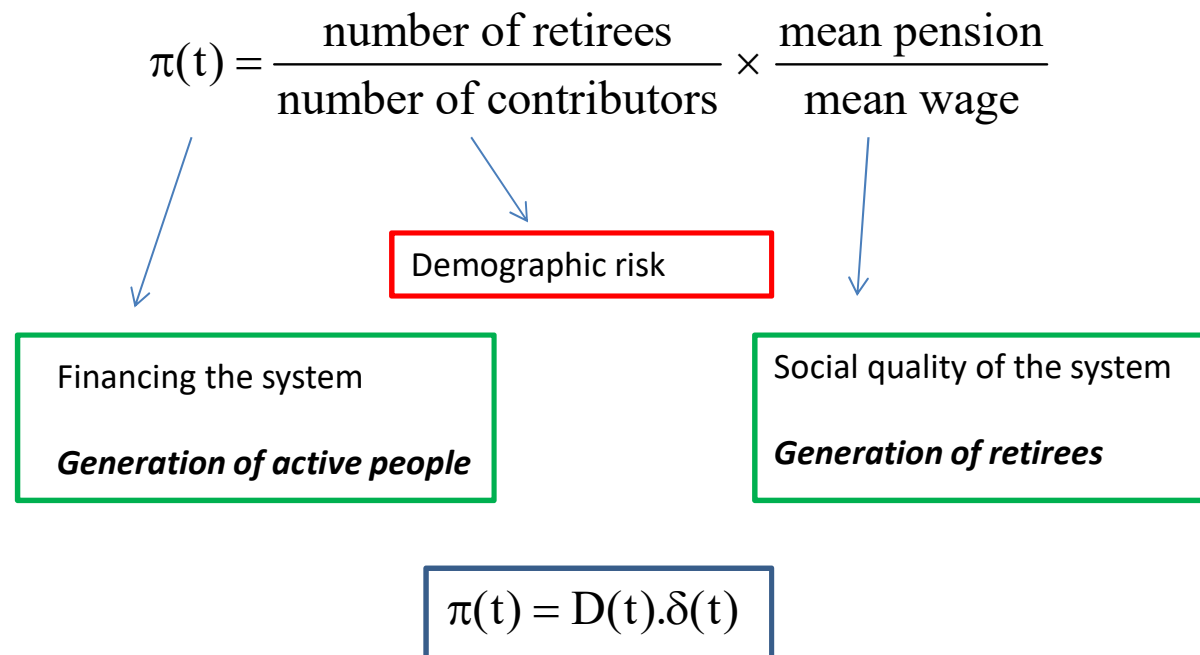


$$D(t) = \frac{B(t)}{A(t)} = \text{dependence ratio}$$



$$\delta(t) = \text{benefit ratio}$$

# PAYG Equilibrium Equation





# Automatic Adjustment Mechanism( AAM)

$$\pi(t) = \frac{B(t)}{A(t)} \cdot \frac{P(t)}{S(t)} = D(t) \cdot \frac{P(t)}{S(t)} = D(t) \cdot \delta(t)$$

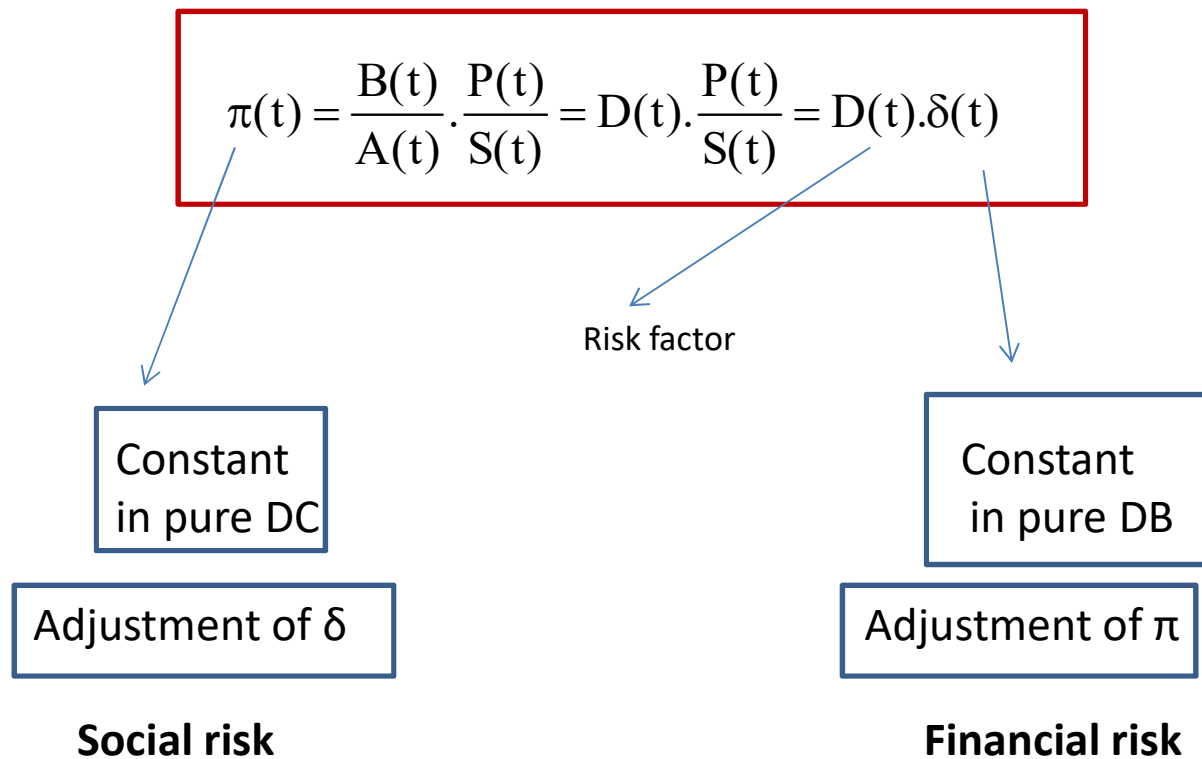
Risk factor


- One equation
- 2 variables

## Automatic Adjustment Mechanism :

In order to maintain financial sustainability  
how to guarantee automatically this equilibrium  
in case of change of  $D(t)$  ( ! *Aging effect* ! )

# The classical answer - DB or DC



- **DB** : risks borne by the active workers  
( *sustainability challenge* )
  - **DC** : risks borne by the retirees  
( *adequacy challenge* )
- 

- **Risk sharing** between generations ?
- Intermediate solutions between DB and DC ?

# Literature


- General principles of Automatic Balance Mechanisms  
*Vidal-Melia et al. (2009)*  
*Boado- Penas et al. (2020)*
- Literature on risk sharing mechanisms in PAYG for given pension architecture  
*Knell(2010)*  
*Godinez-Olivares et al.(2016 a / 2016 b )*  
*Alonso-Garcia et al. (2018a)*  
*Alonso-Garcia /Devolder (2019)*
- Hybrid pension schemes  
*Musgrave (1981)*  
*Schokkaert et al (2020)*

In this paper , special focus on the indexation of pension as a potential automatic balance mechanism

# Our proposal

Sharing the demographic and economic risks between generations in two ways ( double automatic adjustment mechanism )

**LEVEL 1 : sharing the aging risk between workers and retirees**


$$\pi(t) = D(t) \cdot \delta(t)$$

A shock on the dependency ratio will affect jointly the benefit ratio and the contribution rate ( *and not just the benefit ratio (DC) or just the contribution rate( DB)*  ) .

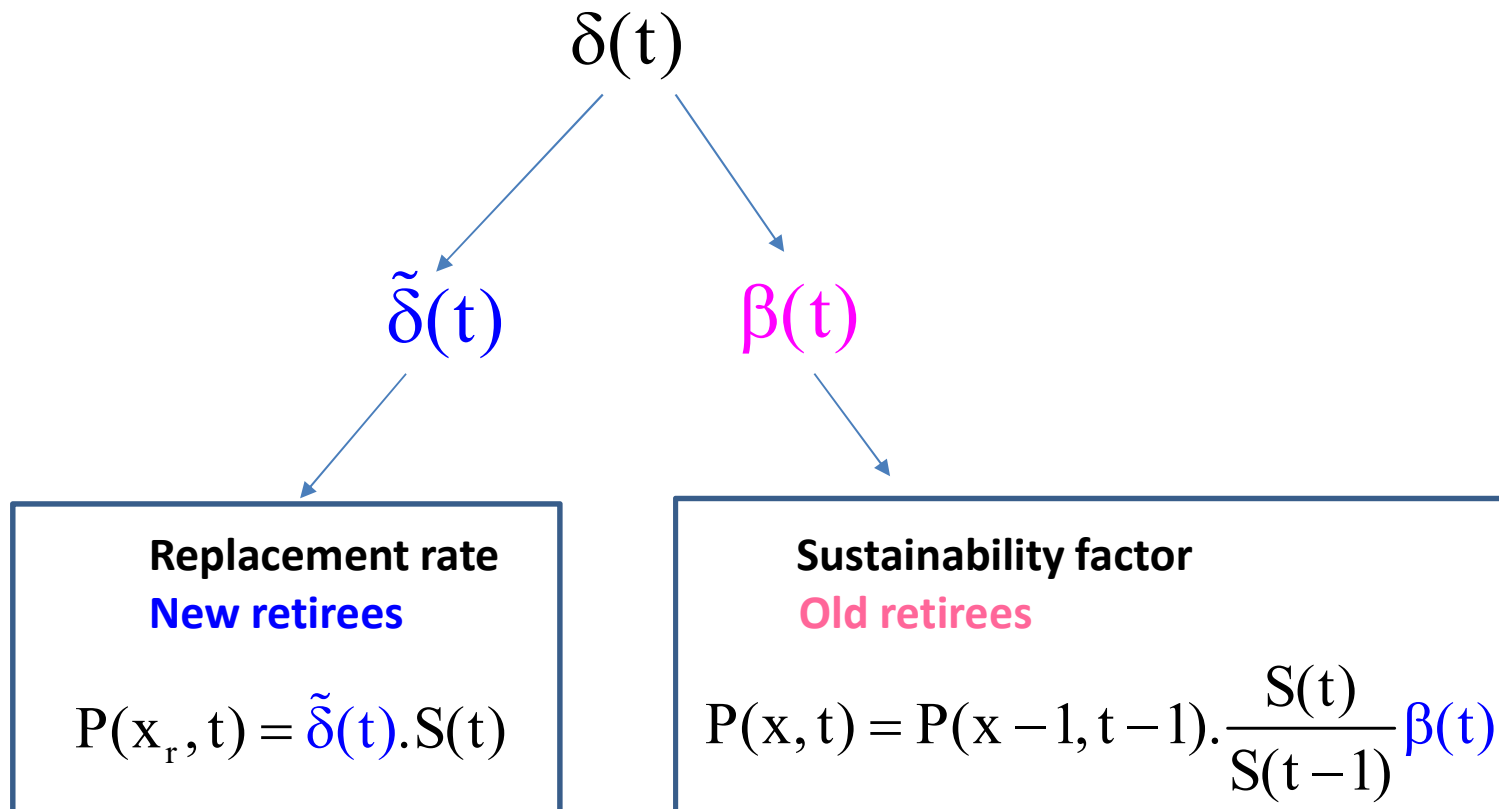
This first level will give us each year a mean benefit ratio  $\delta(t)$

*This mean benefit ratio is the consequence of decisions on pension :*

- *for the new retirees ( first pension / replacement rate)*
- *for the old retirees ( revaluation/indexation of pension of the previous year)*

# Our proposal

**LEVEL 2** : sharing the benefit risk between the new and the old retirees through adapted indexation



# Our proposal

## LINK BETWEEN LEVEL 1 AND LEVEL 2 :

When the sustainability factor and the replacement rate are given ,  
the mean benefit ratio can be computed for a given structure of population

### Relation :

The cross sectional benefit ratio  $\delta(t)$  represents the weighted average of the cohort –specific past replacement rates  $\tilde{\delta}(\cdot)$  modified by the successive sustainability factors  $\beta(\cdot)$

$$\delta(t) = \tilde{\delta}(t).1_{x_r, t} + \sum_{x=x_r+1}^{\omega} \tilde{\delta}(t - (x - x_r)). \left( \prod_{s=t}^{t - (x - (x_r + 1))} \beta(s) \right). 1_{x, t}$$

Density of retirees by age

# Our proposal

## EXTREME CASES / LEVEL 1 :

### Defined Benefit :

$$\delta(t) = \delta_0$$

$$\pi(t) = \delta_0 \cdot D(t)$$

### Defined Contribution :

$$\pi(t) = \pi_0$$

$$\delta(t) = \pi_0 / D(t)$$



# Our proposal

## PARTICULAR CASES / LEVEL 2 :

### Protection of the old retirees :

$\beta(t) = 1$  : full indexation

$\tilde{\delta}(t)$  to be adjusted to fulfill condition on  $\delta(t)$

### Solidarity between all the retirees :

$$\tilde{\delta}(t) = \delta(t)$$

$$\beta(t) = \frac{\delta(t)}{\delta(t-1)}$$

### Protection of the new retirees :

$\tilde{\delta}(t) = \delta_0$  : guaranteed initial replacement rate

$\beta(t)$  to be adjusted to fulfill condition on  $\delta(t)$

## 2. Optimization model

Optimal risk sharing between workers and retirees ( first level )  
and between old and new retirees ( second level)

**Objective function** : penalisation of the distance between the parameters  
and target values ( *quadratic optimization – see Cairns (2000)* )

First level : relative stability of the benefit ratio and the contribution rate  
around equilibrium values

$$\min_{\pi(t), \delta(t)} \left( \rho_t \cdot \left( \frac{\pi(t)}{\bar{\pi}} - 1 \right)^2 + (1 - \rho_t) \cdot D(t) \cdot \left( \frac{\delta(t)}{\bar{\delta}} - 1 \right)^2 \right)$$

Weight-parameter

Democratic proportion  
Between retirees and workers

# Optimal first level

Solution :

$$\delta^*(t) = \bar{\delta}.\bar{\pi}.\frac{\rho_t.\bar{\delta} + (1-\rho_t).\bar{\pi}}{\rho_t.D(t).\bar{\delta}^2 + (1-\rho_t).\bar{\pi}^2}$$
$$\pi^*(t) = \delta^*(t).D(t)$$

Natural choice for the target :

Example : not deviate too much from initial values !

$$\bar{\delta} = \delta_0$$

$$\bar{\pi} = \pi_0$$

# Optimal second level

**Optimal risk sharing** between workers and retirees ( first level )  
**and between old and new retirees ( second level)**

Second level : relative stability of the benefit ratio and the contribution rate around equilibrium values

$$\min_{\pi(t), \delta(t)} \left( \eta_t \cdot \left( \frac{\beta(t)}{\bar{\beta}(t)} - 1 \right)^2 + (1 - \eta_t) \cdot \frac{l_{x_r, t}}{\sum_{x > x_r} l_{x, t}} \cdot \left( \frac{\tilde{\delta}(t)}{\bar{\delta}(t)} - 1 \right)^2 \right)$$

Weight-parameter

Democratic proportion  
between the retirees

## Optimal second level

Solution :

$$\beta^*(t) = \bar{\beta}(t) \frac{\eta_t \bar{\tilde{\delta}}(t)^2 \cdot 1_{x_r t}^2 + (1 - \eta_t) \alpha(t) \cdot \bar{\beta}(t) \cdot D_{x_r, t} (\delta(t) - \bar{\tilde{\delta}}(t) 1_{x_r, t})}{\eta_t \bar{\tilde{\delta}}(t)^2 \cdot 1_{x_r t}^2 + (1 - \eta_t) \alpha^2(t) \cdot \bar{\beta}^2(t) \cdot D_{x_r, t}}$$

$$\tilde{\delta}^*(t) = \bar{\tilde{\delta}}(t) \frac{\eta_t \bar{\tilde{\delta}}(t) \cdot 1_{x_r t} (\delta(t) - \alpha(t) \cdot \bar{\beta}(t)) + (1 - \eta_t) \alpha^2(t) \cdot \bar{\beta}^2(t) \cdot D_{x_r, t}}{\eta_t \bar{\tilde{\delta}}(t)^2 \cdot 1_{x_r t}^2 + (1 - \eta_t) \alpha^2(t) \cdot \bar{\beta}^2(t) \cdot D_{x_r, t}}$$

$$\text{with : } \alpha(t) = \sum_{x=x_r+1}^{\omega} \tilde{\delta}(t - (x - x_r)) \cdot \left( \prod_{s=t}^{t - (x - (x_r + 1))} \beta(s) \right) \cdot 1_{x, t}$$

Natural choice for the target

$$\bar{\tilde{\delta}}_t = \delta_t \quad (\text{solidarity between retirees})$$

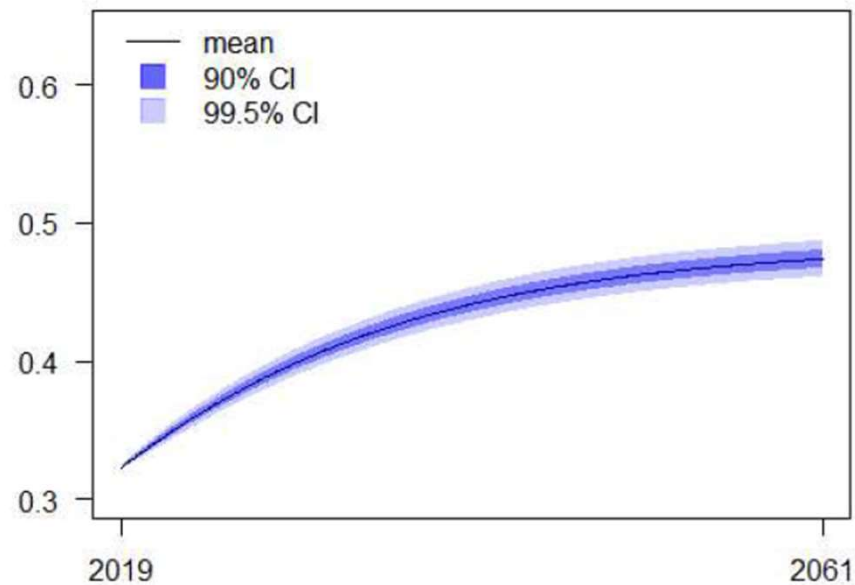
$$\bar{\beta}_t = 1 \quad (\text{full indexation})$$

### 3. Numerical illustration

#### Evolution of the dependency ratio

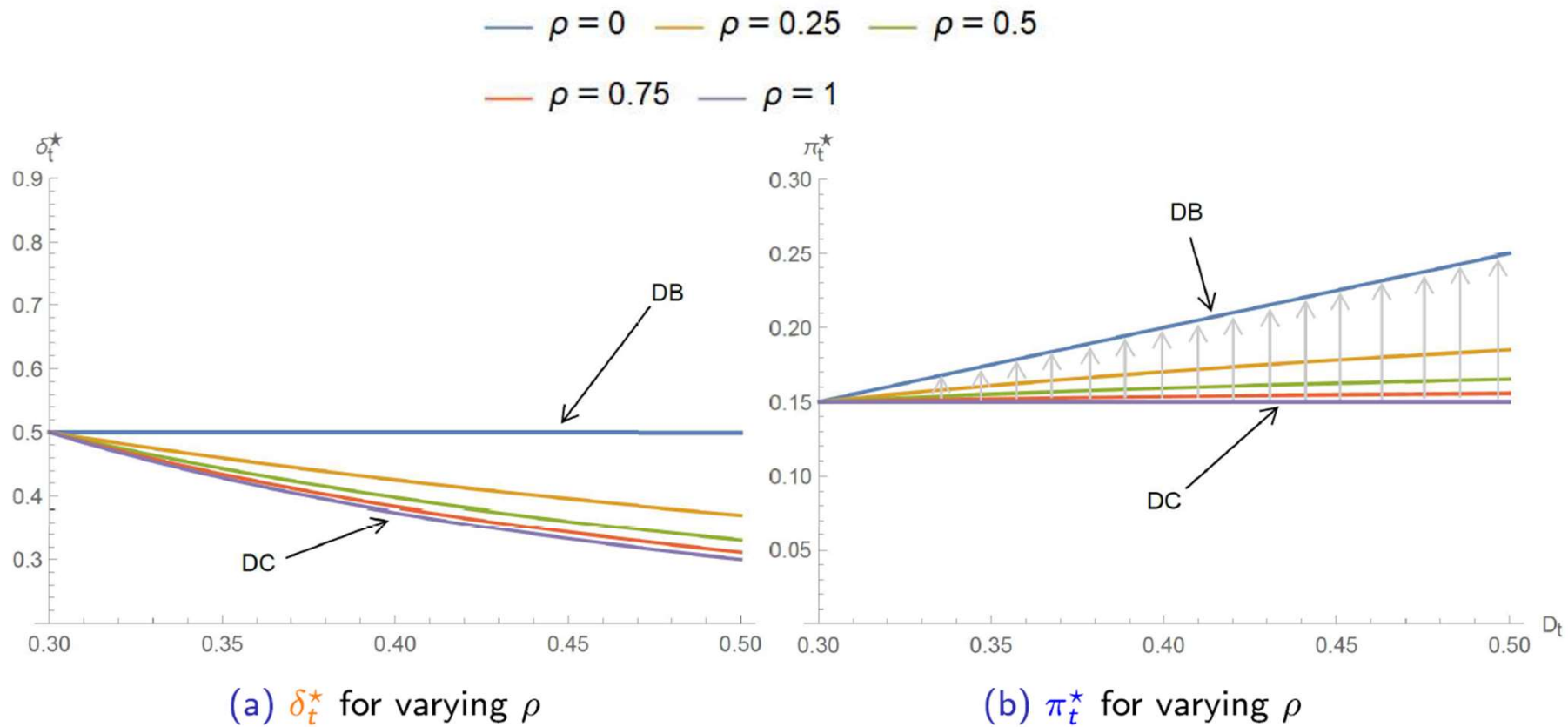
Black- Karasinsky model

$$d \ln(D(t)) = \alpha.(\ln(D_{\infty}) - \ln(D(t))) dt + \sigma dw(t)$$



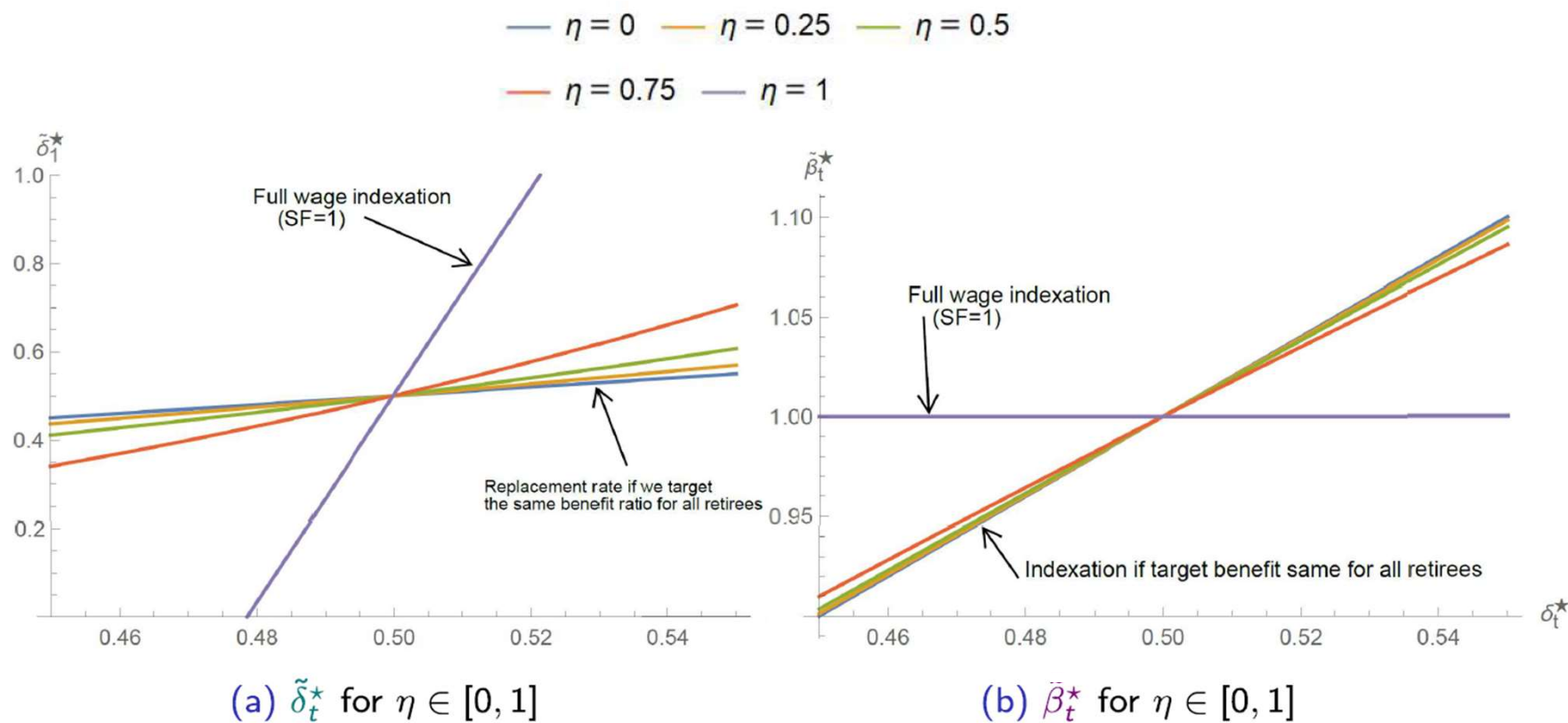
# Numerical illustration

First level : between DB and DC



# Numerical illustration

## Second level : indexation vs replacement rate





# Conclusion

- As soon as the planner takes simultaneously into account the interest of the workers and of the retirees, **neither DB nor DC is optimal**

Hybrid pension plans must be preferred ( intergenerational risk sharing of the aging cost )

- In an aging phase, **Fully indexing pensions to salaries is unsustainable** and could yield either to unreasonable contribution rates or to low and even negative replacement rates for new retirees !

A sustainability factor on indexation ( partial indexation ) must be preferred

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**THANK YOU !!!!**

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